Optimization of Investment and Storage Constrained Multi-Item Inventory Model with Quality and Environmental Concern

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***Abstract* - This paper has introduced the EOQ in deterministic stock model of multi-item when the target capacity is subjected to venture and limit of deficiency space limitations. The requirements are thought to be dynamic if the left hand side does not fulfil the right hand side condition. In this way, Lagrange Technique is utilized to locate the new multi-item EOQ in every four models with every limitation to accomplish the new model of EOQ. In order to provide good quality of items screening cost is included. Additionally in consideration of environmental issues disposal cost is included to make the model into practical application. Numerical example is provided to validate the model.**

***Keywords* - Inventory, constraint, storage, investment, screening.**

I. INTRODUCTION

The inventory control has the capacity of coordinating the products through the whole manufacturing cycle from the demanding of materials to the stock of completing merchandise in a deliberate way to meet the targets of most extreme profit with least venture and proficient plant operation. Inventory level has different limitations, for example, restricted inventory space, constrained spending plan accessible for stock, level of administration consideration towards each item in the inventory. Numerous researchers have concentrated on the stocks model or inventory model with constrained inventory storage and speculation.

Teng and Yang [7] proposed a deterministic inventory lot size models with time-varying demand and generalized holding cost. Abou-El-Ala and Kotb [10] applied geometric programming approach to solve some inventory models with variable inventory costs. Shawky and Abou-El-Ala [1] solved a constrained production lotsize model with trade policy by geometric programming and Lagrange methods. Other such inventory models were developed by Cheng [16]. Jung and Klein [5] and Mandal et al [12]. Recently Kotb [8] discussed multi-item EOQ model with limited storage space and setup cost constraints with varying holding cost. The obliged multi-item stock model has been dealt by numerous researchers. Maloney and Klein [2] investigated obliged multi-item inventory framework. Abou-El-Ala and Kotb [10] builtup on fresh inventory model under two limitations. Ben-Daya and Abdul Raouf [9] talked about the issue of inventory model including lead time as choice variables.

The idea of fuzzy non-linear programming is utilized to discover the development of EOQ model under limited space and spending plan. The optimal replenishment strategy has researches for multi-item EOQ under terms of admissible instalment of postponement, requirement of a financial plan. It is more conservative for the buyers to give the entire instalment notwithstanding when the earned interest is more prominent than the charged interest. Other researchers created two deterministic EPQ models for break down items which distributed Weibull with interest rate. They expected the finite production rate is proportionate with the time ward of mean interest and the cost generation unit is contrarily equivalent to the rate of the production [11]. For a long time, the issue of traditional EOQ talked about with assortment forms and determined by numerous analysts in book references, articles. It begins from a basic model of single-item with related interest to unit cost until multi-item with numerous imperatives.

Therefore, in this study, for advancing practical use in a real world, we propose an inventory model with good quality products under screening and disposal cost to reduce environmental damage. In addition the amount of profit that could be generated from interest payments on a given amount of money if it were invested in government bonds is the cost of money. This criterion is under concern in this model.

This paper means to develop an inventory model to estimate the EOQ multi-item when the target capacity is subjected to venture and limit of lack space imperatives under quality and environmental concern. This condition can be accomplished when the left hand side of the imperative is not fulfilling the right hand side, as such; the left hand side is more prominent than the right hand side which means the requirement is dynamic.

The paper is organized as follow: Section 2 elucidates the Assumptions and Notations used in this model. Section 3 formulates the mathematical model. In the application part section 4 explains the model with numerical example and finally section 5 concludes the proposed work.

II. ASSUMPTIONS

The following assumptions are made to develop the model. The replenishment is instantaneous for model 1 and the replenishment is gradual for model 2.

The demand is constant and known.
Lead time is equal to zero.
Shortage is allowed.

2.1. Notations

Q - Ordering quantity
Z - Total inventory cost
K - Setup cost
h - Holding cost (h = I * C)
C - Unit cost
I - inventory holding cost percentage (%)
p - Cost of shortage
β - Demand rate per unit
R - Production rate
S - Maximum shortage quantity
y - Screening cost
H - Freight cost per delivery
C_s - unit disposal cost of scrap items
q - Percentage of scrap items in defectives
α - Percentage of defective items in Q
T - Inventory cycle length
FV - Future value
r - Interest

III. MATHEMATICAL MODEL

The deterministic models in inventory control are divided into four models. They are multi-item deterministic inventory models under purchase and production classification with investment and storage constraint.

Here the decision variables are order quantity investment constraints λ and storage constraint f. The multi-item inventory model notations are

Q_j - order quantity for each item j = 1, 2, . . . , n.
Z_j - the total cost for each item j = 1, 2, . . . , n.
A_1 - maximum value of investment
A_2 - Capacity of storage space allowed for each items, j = 1, 2, . . . , n.
c_j - the investment for each item, j = 1, 2, . . . , n.
f_j - the required space for each item, j = 1, 2, . . . , n.

Investment constraint, $\sum_{j=1}^{n} c_j Q_j \le A_1$

Capacity of storage space constraint, $\sum_{i=1}^{n} f_j Q_j \le A_2$

The optimal value is found by using Lagrange Method.

3.1. Model 1 with investment constraint

The total cost is composed for setup cost, holding cost, shortage cost, screening cost, freight cost, disposal cost, and cost of money and investment constraint.

$$L(Q,\lambda) = \frac{K_j B_j}{Q_j} + \frac{h_j (Q_j - S)^2}{2Q_j} + \frac{PjS^2}{2Qj} + y\beta_j + \frac{H\beta_j}{Q_j} + Csq_j\alpha_j\beta_j + \frac{FV\beta_j}{Q_j(Hr)} - \lambda\left(\sum_{j=1}^{n} C_j Q_j - A_1\right) \quad \ldots (1)$$

$$\frac{\partial L}{\partial Q} = \frac{-K_j B_j}{Q_j^2} + \frac{h_j}{2}\left(\frac{2(Q_j - S)Q_j(Q_j - S)^2}{Q_j^2}\right) - \frac{P_j S^2}{2Q_j^2} - \frac{H\beta_j}{Q_j^2} - \frac{FV\beta_j}{Q_j^2(Hr)} - \lambda C_j = 0$$

$$\Rightarrow -K_j\beta_j + \frac{h_j}{2}\left(2(Q_j - S)Q_j(Q_j - S)^2\right) - \frac{P_j S^2}{2} - H\beta_j - \frac{FV\beta_j}{(1+r)} - \lambda C_j Q_j^2 = 0$$

$$h_j Q_j^2 - 2\lambda C_j Q_j^2 = 2\left(K_j\beta_j + H\beta_j + \frac{FV\beta_j}{(1+r)}\right) + S^2(P_j + h_j)$$

$$Q_j^* = \sqrt{\frac{2\beta_j\left(K_j + H + \frac{FV}{(1+r)}\right) + S^2(P_j + h_j)}{h_j - 2\lambda C_j}} \quad \ldots (2)$$

Now we have to find the value of λ. Trial and error method is possible but for accurate result we derive the value λ as follows:

$$\sum_{j=1}^{n} C_j \sqrt{\frac{2\beta_j\left(K_j + H + \frac{FV}{(1+r)}\right) + S^2(P_j + h_j)}{h_j - 2\lambda C_j}} = A_1$$

Here $h_j = I_j \times C_j$

$$\sum_{j=1}^{n} C_j \sqrt{\frac{2\beta_j\left(K_j + H + \frac{FV}{(1+r)}\right) + S^2(P_j + I_j C_j)}{I_j C_j - 2\lambda C_j}} = A_1$$

$$\sum_{j=1}^{n} \sqrt{\frac{2\beta_j\left(K_j + H + \frac{FV}{(1+r)}\right) + S^2(P_j + C_j)}{2\lambda}} = A_1$$

$$\sqrt{2\lambda} = \frac{1}{A_1}\sqrt{2\beta_j\left(K_j + H + \frac{FV}{(1+r)} + S^2(P_j + C_j)\right)}$$

$$\lambda = -\frac{\left(\frac{1}{A_1^2}\sqrt{2\beta_j\left(K_j + H + \frac{FV}{(1+r)}\right) + S^2(P_j + C_j)}\right)^2}{2} \quad \ldots (3)$$

3.2. Model 1(a) with capacity of storage space constraint

Using similar approach in investment constraint this model is developed.

$$L(Q,\lambda) = \frac{K_j B_j}{Q_j} + \frac{h_j (Q_j - S)^2}{2Q_j} + \frac{PjS^2}{2Qj} + y\beta_j + \frac{H\beta_j}{Q_j} + Csq_j\alpha_j\beta_j + \frac{FV\beta_j}{Q_j(Hr)} - \lambda\left(\sum_{j=1}^{n} f_j Q_j - A_2\right) \quad \ldots (4)$$

$$\frac{\partial L}{\partial Q} = \frac{-K_j B_j}{Q_j^2} + \frac{h_j}{2}\left(\frac{2(Q_j - S)Q_j(Q_j - S)^2}{Q_j^2}\right)$$

$$-\frac{P_jS^2}{2Q_j^2}-\frac{H\beta_j}{Q_j^2}-\frac{FV\beta_j}{Q_j^2(Hr)}-\lambda f_j=0$$

We get the optimal order quantity as

$$Q_j^* = \frac{2\beta_j\left(K_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+h_j)}{h_j-2\lambda f_j} \quad \ldots (5)$$

3.3. Model (2) with investment constraint

This is similar to model (1) but this model is based on production quantity where model (1) is dealing with purchases.

$$L(Q,\lambda) = \frac{K_jB_j}{Q_j}+\frac{h_j(Q_jb_j-S)^2}{2Q_jb_j}+\frac{PjS^2}{2Qj}+y\beta_j+\frac{H\beta_j}{Q_j}$$

$$+Csq_j\alpha_j\beta_j+\frac{FV\beta_j}{Q_j(1+r)}-\lambda\left(\sum_{j=1}^{n}C_jQ_j-A_1\right) \quad \ldots (6)$$

Here $\frac{\partial L}{\partial \lambda} = b_1 - C_jQ_j = 0$

$$\frac{\partial L}{\partial Q} = \frac{-K_jB_j}{Q_j^2}-\frac{h_j}{2}\left(\frac{2(Q_jb_j-S)b_jQ_j(Q_jb_j-S)^2}{Q_j^2b_j^2}\right)$$

$$-\frac{P_jS^2}{2Q_j^2b_j}-\frac{H\beta_j}{Q_j^2b_j}-\frac{FV\beta_j}{Q_j^2b_j(1+r)}-\lambda C_j=0$$

$\Rightarrow$

$$h_jQ_j^2b_j^2-h_jS^2-2\lambda C_jQ_j^2b_j = 2K_j\beta_jb_j+2\beta_jH+\frac{FV(2B_j)}{(1+r)}+S^2P_j$$

$$\therefore Q_j^* = \sqrt{\frac{2\beta_j\left(K_jb_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+h_j)}{h_jb_j-2\lambda C_j}} \quad \ldots (7)$$

To find λ, $\sum_{j=1}^{n}C_j\sqrt{\frac{2\beta_j\left(K_jb_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+h_j)}{b_jh_j-2\lambda C_j}} = A_1$

where $h_j = I_j * C_j$

$$\sqrt{b_j-2\lambda} = \frac{1}{A_1}\sqrt{2\beta_j\left(K_jb_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+C_j)}$$

$$\lambda = -\frac{\left(\frac{1}{A_1^2}\sqrt{2\beta_j\left(K_jb_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+C_j)}\right)^2}{2}-b_j \quad \ldots (8)$$

3.4. Model (2a) with capacity of storage space constraint

$$L(Q,\lambda) = \frac{K_jB_j}{Q_j}+h_j\frac{(Q_jb_j-S)^2}{2Q_jb_j}+\frac{P_jS^2}{2Q_jb_j}+y\beta_j+\frac{H\beta_j}{Q_jb_j}$$

$$+Csq_j\alpha_j\beta_j+\frac{FV\beta_j}{Q_jb_j(1+r)}-\lambda\left(\sum_{j=1}^{n}f_jQ_j-A_2\right) \ldots (9)$$

$$\frac{\partial L}{\partial Q} = \frac{-K_jB_j}{Q_j^2}+\frac{h_j}{2b_j}\left(\frac{2Q_j(Q_jb_j-S)b_j-(Q_jb_j-S)^2}{Q_j^2}\right)$$

$$-\frac{P_jS^2}{2Q_j^2b_j}-\frac{H\beta_j}{Q_j^2b_j}-\frac{FV\beta_j}{Q_j^2b_j(1+r)}-\lambda f_j=0$$

$\frac{\partial L}{\partial \lambda} = b_1 - f_jQ_j = 0$

Finally we get the optimal order quantity as

$$Q_j^* = \sqrt{\frac{2\beta_j\left(K_jb_j+H+\frac{FV}{(1+r)}\right)+S^2(P_j+h_j)}{h_jb_j-2\lambda f_jb_j}} \quad \ldots (10)$$

IV. NUMERICAL EXAMPLE

To illustrate the above model following values are considered.

No. of items - 1, 2, 3
β - 33, 24, 20
K_j - 25, 18, 20
h_j - 1, 1.5, 2
C_j - 1, 0.5, 2.5
P_j - 3, 6, 8
S_j - 14, 13, 12
H = 10, FV = 20, r = 3
Total allowed investment – 145

Using Q^* - Order quantity (optimal) equation as $\lambda = 0$ we get $Q_1^* = 58.5$, $Q_2^* = 43.6$, $Q_3^* = 37.6$

where $\sum_{j=1}^{3}C_jQ_j \le A_1 \Rightarrow 174.3 \le 145$. The constraint is not satisfied. New value of Lambda is $\lambda = -0.231$.

Items	Qj/week
1	48.39
2	40.58
3	30.00

$\sum_{j=1}^{3}C_jQ_j = 143.68 \le A_1 = 145.$

using the total allowable investment we reached an optimal value which improves the profit for the vendor.

V. CONCLUSION

This paper has created and investigated two deterministic inventory models when they are subjected to speculation and limit of deficiency space limitations to locate the new mathematical statement or estimation of EOQ. The numeric points of preference of utilizing these comparisons are imperative and vital when we can't surpass the aggregate speculation and the aggregate permitted storage area.

In this way, we should separate these new mathematical models of the EOQ per optimum exploitation of the investment and capacity zone additionally don't surpass these two limitations. The discoveries demonstrate that, the two requirements with every inventory models that are introduced will help the firm to locate the optimal EOQ in every model using Lagrange technique. Numerical illustration clearly presents the optimal value of the model presented.

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