

Speech Recognition Using Hidden Markov Model

Ankita Gupta*

Gajendra singh**

*-M.Tech(communication & Information).

Deptt. of Electronics & Communication, SSITM,

Aligarh

Email:-ankita.gupta.ssitm.20@gmail.com

** H.O.D

Dept. of Electronics & communication , SSITM

Neha Sharma#,

#-Assistant Lecturer

Dept. of Electronics And Communication

ACET Aligarh

Email:-nsnehasharma14@gmail.com

Abstract-The aim of this paper on “Speech Recognition using Hidden Markov Model (HMM)” is to create and develop a limited vocabulary speech recognition system with the help of HMM to statically model the words in the dictionary. For this purpose we use a database that contains a samples of different speaker (for better accuracy, at last twenty speakers) are desirable. So, this speech recognition system is speaker dependent to test the theory presented in this paper and gets a practical experience of speech recognition using HMM the speech recognizer machine should be implemented on Matlab. This implementation is based on mathematical calculations and equation. Using these implementations, the different word samples spoken by different speakers undergoes through different sub process like segmentation of signal, clustering of signal. Then its training phase described in this paper to get a reliable training at least 20 samples of each word from 20 different speakers are required to record in an environment free from external disturbance. With the help of these information's, a speech recognition model is ready to recognize the given speech.

KEY WORDS

HMMS – Hidden Markov Model

INTRODUCTION

The purpose of this paper on speech recognition using HMM is to get a deeper theoretical and practical understanding of speech recognizer. The work started by examine a currently existing state of speech recognizer. The work started by examine a currently existing state of speech recognizer called Hidden Markov Tool Kit (HTK) HTK is not easy to use, because it need to have its complete theoretical knowledge for its functioning with the experience from HTK, it was desired to make a system that would be easier to use. The web page is limited because it is not possible to try different settings. This is why we use a programming language Matlab for this purpose because it is easily extended with new features applying this knowledge in practical manner speech recognizer implemented in Matlab is ready to use.

What is speech recognition?

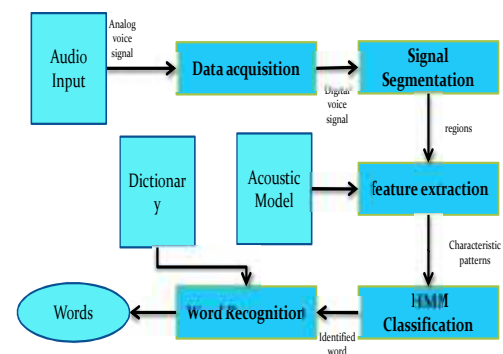
Speech recognition is the process of recognizing some specific

characteristic features of an audible signal.



Figure (a): Basic Speech Recognition Process

Speech Recognition Engine



In speech recognition Engine, we have a caustic model which produce the cepstral coefficients & match that word with the trained HMM words dictionary to identify the test word a recent preponderance of recognizers based on statistical models, It was felt that a HMM clustering model would be more effective than a template based approach. A HMM based recognizer first processes the speech signal to convert it into digital signal, then it performs segmentation of this digital signal to derive its

cepstral coefficients (characteristic feature of a signal) then after its training phase a dictionary of words is ready to recognize any test word.

What is HMM?

A Hidden Markov Model is a finite state machine having a fixed number of states in which Transition of states takes place from one state to another and also the next state only depends on present state. It is a finite set of states, with a probability distribution. The transition among the states is governed by transition probabilities. An outcome can be generated according to distribution. Only the outcome not the state is visible to an external observer. It is a statistical method of characterizing the spectral properties of frames of a pattern. The underlying assumption of HMM is that the speech can be well characterized as parametric random process and that the parameters of the stochastic process can be estimated in well defined manner. A HMM is characterized by three matrices i.e. A, B,

A → State Transition Probability matrix ($N \times N$)

B → Observation Probability Distribution Matrix ($N \times M$)

→ Initial state Distribution Matrix ($N \times 1$)

where, N = Number of states in HMM

M = Number of observation symbols.

Therefore HMM can be defined as.

$N, M = (A, B, \pi)$

Where → Trained HMM

Three Basic Issues For HMMs

In order to apply HMMs for speech recognition, we need to address three problems stated below

1- Evaluation Problem

Given the observation sequence $O = \{O_1, O_2, \dots, O_T\}$

How do we efficiently compute $P\{O/\}$

Solution : Recursive procedures like forward and Backward Procedure exist which can compute $p\{O/\}$

- Define forward variable

$$\alpha_t(i) = P\{O_1, O_2, \dots, O_t, q_t = i / \}$$
 recursion (1)

$$\alpha_{t+1}(i) = b_j(O_{t+1}, i) (\sum_l \alpha_t(l) a_{lj})$$

- Similarly backward variable

$$\beta_t(i) = P\{O_{t+1}, O_{t+2}, \dots, O_T, q_t = i / \}$$
 recursion (2)

$$\beta_t(i) = (\sum_j \beta_{t+1}(j) a_{ij} b_j(O_{t+1}))$$
 (3)

Initialising :

$$\alpha_1(j) = \pi_j b_j(O_1) \text{ and } \beta_t(i) = 1$$
 (4)

$$\text{Hence, } P\{O/\} = \sum_i \alpha_t(i) \beta_t(i)$$
 (5)

2- Decoding Problem

Given the observation sequence $O = \{O_1, O_2, \dots, O_T\}$ and a model λ , how do we choose a corresponding state sequence $Q = \{q_1, q_2, \dots, q_T\}$ which is optimal in some meaningful sense.

Solution- Viterbi Algorithm finds the single best sequence or for given observation sequence O. The following equation are presented which is viterbi Algorithm.

a) Initialization:

$$\delta_1(i) = \pi_i b_i(O_1), 1 \leq i \leq N$$
 (6)

$$\delta_1(i) = 0$$
 (7)

b) Recursion :

$$\delta_t(i) = \max_{1 \leq j \leq N} [\delta_{t-1}(j) a_{ji}] b_j(O_t), 2 \leq t \leq T$$
 (8)

$$i_t(i) = \arg \max_{1 \leq i \leq N} [\delta_{t-1}(i) a_{ij}], 2 \leq t \leq T$$
 (9)

c) Termination :

$$P^* = \max_{1 \leq i \leq N} [\delta_T(i)]$$
 (10)

$$q^*_t = \arg \max_{1 \leq i \leq N} [\delta_t(i)]$$
 (11)

d) Path (State sequence) back tracking :

$$q^*_{t-1} = \arg \max_{1 \leq i \leq N} [\delta_{t-1}(i) a_{iq^*_t}], t = T-1, T-2, \dots, 1$$
 (12)

3- Learning Problem

This is the problem of parameter estimation. This is by far the toughest problem of HMM. How do we adjust the model parameter.

$\lambda = (A, B, \pi)$ to maximize $P(O/\lambda)$

Solution- We can, however, choose

$\lambda = (A, B, \pi)$ such that probability $p(O/\lambda)$ is locally maximized using iterative procedure such as Baum-welch method. But Baum Welch re-estimation procedure suffers from following problems-

- Numerical problem and hence hard to Implement.
- Needs special scaling.
- Needs multiple observation sequences. There exist one more method which can overcome these problems & can be used to train a HMM. This is segmental K-means method.

Segmental k-means uses the solution to problem 2 to modify the model parameters. This process undergoes following steps.

- Consider a starting HMM
- Calculate Transition Probability matrix A
- Calculate Emission matrix or observation symbol probability Distribution matrix B.
- Update HMM parameters according to Transition & Emission probability)
- Continue recursion & update the parameters of HMM.

Applications

Speech recognition using the technique Hidden markov Model is very useful in many applications and environments in our daily life. Generally speech recognizer is a machine which understands human and their spoken words in some way & can act thereafter.

It can be used in following fields-

- 1- In car environment to voice control non-critical operations, such as dialing any phone number.
- 2- On board Navigation, presenting the driving route to driver.
- 3- By applying voice control, traffic safety will be increased.
- 4- Facilitate a person with functional disability or other kinds of handicaps. To make their daily works easier with their voice they could operate the light switch, turn off/ On any domestic appliances. This leads to intelligent homes where these operations can be made available for common man as well as for handicapped.
- 5- Can be used in Hand held digital recorder.
- 6- This technique can be useful in any field whether it is commercial, Industrial, Military, or forensic applications.

RESULT

To test the theory presented in this paper and get a practical experience of speech recognizer is to be implemented on Matlab. The implementation is based on mathematical calculations and equation that are explained as the three Basic problems of HMMs. (i.e. Evaluation problem, Decoding Problem, Learning Problem) Using these implementations HMMs for different words & speech can be designed or trained. In complete process to training a HMM different words spoke by several speaker undergoes through different sub processes like segmentation, derivation of cepstral coefficients then clustering and after that training of HMMs of different speech or word. For the above processes, we implement the whole process on Matlab because all these process and functions are predefined on Matlab. Therefore we need not to go in deep and large mathematical calculations. Once the HMMs are trained speech recognizer machine is ready to recognize any test speech. To get a reliable training at least twenty samples of different words from at least twenty speakers are required to record in a disturbance free environment. The training results in speech recognizer mode for different speech. The model represents a static mode (Hidden Markov Model, HMM) for each word.

HMM Performance on Isolated Word Recognition

We conclude this section on isolated word recognition using HMMs by giving a set of performance results (in terms of average word error rate) on the task of recognizing isolated digits in a speaker independent manner. For this task, a training set consisting of 100 occurrences of each digit by 100 talkers (i.e., a single occurrence of each digit per talker) was used. Half the talkers were male; half female. For testing the algorithm, we used the initial training set, as well as three other independent test sets with the following characteristics:

- TS2 : the same 100 talkers as were used in the training; 100 occurrences of each digit
- TS3 : a new set of 100 talkers (50 male, 50 female); 100 occurrences of each digit
- TS4 : another new set of 100 talkers (50 male, 50 female); 100 occurrences of each digit

The results of the recognition tests are given in Table 1. The recognizers are the following:

- LPC/DTW : Conventional template-based recognizer using dynamic time warping (DTW) alignment
- LPC/DTW/VQ : Conventional recognizer with vector quantization of the feature vectors ($M = 64$)
- HMM/VQ : HMM recognizer with $M = 64$ codebook
- HMM/CD : HMM recognizer using continuous density model with $M = 5$ mixtures per state
- HMM/AR : HMM recognizer using autoregressive observation density

Table 5.2 Average Digit Error Rates for Several Recognizers and Evaluation Sets

Recognizer Type	Evaluation Set			
	Original Training	TS2	TS3	TS4
LPC/DTW	0.1	0.2	2.0	1.1
LPC/DTW/VQ	–	3.5	–	–
HMM/VQ	–	3.7	–	–
HMM/CD	0	0.2	1.3	1.8
HMM/AR	0.3	1.8	3.4	4.1

It can be seen that, when using a VQ, the performance of the isolated word recognizer degrades in both the conventional and HMM modes. It can also be seen that the performances of the conventional template-based recognizer, and the HMM recognizer with a continuous density model are comparable. Finally Table 1 shows that the autoregressive density HMM gives poorer performance than the standard mixture density model.

CONCLUSION

In this paper we have to present the theory of hidden Markov models in simple ways and concepts. Our main purpose is to focus on physical explanations of basic mathematics, therefore. We have avoided long drawn out proofs and derivations of the key results and to focus primarily on trying to interpret the meaning of math and its practical implementation in real world systems. We have also illustrated some of the applications of theory of HMMs to simple problems in speech recognition, and point out how the technique could be applied to more advanced speech recognition problems.

For understanding the fundamentals of speech recognition using hidden Markov model, in real sense, a person needs hands on experience with the software, hardware and platform. Hence strongly encourage all serious readers of this paper to program the algorithms, implement the system, and literally build application.

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Without practical experience reader will not find the words written in this paper alive.

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