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Analysis of a Fuzzy Queueing System with a Removable and Non-Reliable Server

W. Ritha[#1], Sreelekha Menon.B[#2]

[#1]*Department of Mathematics, Holy Cross College, Tiruchirappalli- 620 002, Tamilnadu, India.*

ritha_prakash@yahoo.co.in

[#2]*Department of Mathematics, SCMS School of Engineering and Technology, Kerala - 683 582, India.*

Abstract - **This paper proposes a procedure to construct the membership function of the performance measures in a queueing system with removable and reliable server in which the arrival rate and service rate are fuzzy numbers. The basic idea is to transform the fuzzy que with removable and reliable server to a family of conventional crisp que with α-cut approach. By using Zadeh's extension principle a pair of mixed integer non-linear programs are developed to calculate the upper and lower bounds of the system performance measures at possibility α. Since the performance measures are expressed by membership functions rather than by crisp values they completely conserve the fuzziness input information when some ideas are ambiguous.**

Keywords - ***Removable server, non-reliable server, mixed integer non-linear programming, α-cut.***

I. INTRODUCTION

The queueing model under consideration is one with removable and non-reliable server that operates with an N policy. The term removable server is just an abbreviation of the system for the system of turning on or turning off the server, depending on the number of customers in the system. A non-reliable server means that the server is typically subjected to unpredictable breakdowns. Here we consider the operational characteristics of an M/G/1 que in which ; turn the server on whenever $N (N \geq 1)$ or more customers are present, turn the server off only when no customers are present. After the server is turned off, the server may not operate until N customers are present in the system. Thus we consider a queuing system in which the server is removable and applies N policy.

Several research works have been conducted on reliable as well as non-reliable servers with N policy. Yadin and Naor (1963) were the first who developed an N policy M/M/1 queuing system for a reliable server. For a reliable server, M/G/1 queuing system was developed by several researchers such as Bell (1971, 1972), Heyman (1968), Kimura (1981), Teghen (1987), Tijms (1986), Artalejo (1998), Wang and Ke (2000) and so on. Avi-itzhak and Naor (1963) studied the M/M/1 queuing system for a non-reliable server where the service rule does not depend on the number of customers in queue.

Existing research works including those mentioned above, have been developed to search for the characteristics of the queuing model when the arrival and service patterns are known exactly. However in many real world applications, these parameters may not be estimated precisely. In such situation queuing models with fuzzy parameters is much more useful and realistic than the commonly used crisp queues. In crisp queues membership functions of the performance measures are not completely described. If we can drive the membership function of some performance measures we obtain a more reasonable and realistic performance measure because it maintains the fuzziness of input information that can be used to represent the fuzzy system more accurately.

In this paper, we develop a method to calculate the fuzzy performance measures with arrival rate and service rate are fuzzy numbers. We will apply Zadeh's extension principle to transform fuzzy FM/FG/1 queue to crisp M/G/1 queue. As the α value varies crisp queues are described and solved using a pair of mixed integer non-linear programming (MINLP) techniques. The solution from MINLP constructs the membership functions of interest.

II. MODEL DESCRIPTION

Consider a queuing system in which customers arrive at a single server facility as a Poisson process with arrival rate $\tilde{\lambda}$, where $\tilde{\lambda}$ is a fuzzy number and the arriving customers are served in the order of their arrivals (FCFS). The service time constitute a set of independent and identically distributed random variables with a common distribution function $F_S(S)$; a mean $\tilde{\mu}_S$ and a finite variance $\tilde{\sigma}_R^2$, where $\tilde{\mu}_S$ and $\tilde{\sigma}_R^2$ are fuzzy numbers. The idle period starts when the system is empty. When the server is working it may breakdown at any time with a fuzzy Poisson breakdown rate $\tilde{q}$. When the server fails it is immediately repaired in a repair facility, where the repair times are independent and identically distributed random variables with a common distribution function $F_R(t)$; a fuzzy mean $\tilde{\mu}_R$ and a fuzzy variance $\tilde{\sigma}_R^2$. When the repair is complete the server immediately returns for service.

We define the expected number of customers in the system for M/G/1 queuing system under N policy with server breakdowns as $E(N_S)$. From Wang and Ke (2002) we have

$$E(N_S) = \frac{N-1}{2} + \rho(1+q\mu_R) + \frac{\lambda^2\left[(1+q\mu_R)^2(\mu_S^2+\sigma_S^2)+q\mu_S(\mu_R^2+\sigma_R^2)\right]}{2\left[1-\rho(1+q\mu_R)\right]}$$

where $\rho = \lambda\mu_S$.

In the Queuing Model (FM/FG/1) the arrival rate, service rate, breakdown rate and variance ; which are fuzzy numbers ; are approximately known and can be represented by a convex fuzzy set. A fuzzy set $\tilde{A}$ in its universal set Z is convex if $\mu_{\tilde{A}}(\phi z_1 + (1-\phi)z_2) \geq \min\{\mu_{\tilde{A}}(z_1), \mu_{\tilde{A}}(z_2)\}$ where $\mu_{\tilde{A}}$ is its membership function, $\phi \in [0, 1]$ and $z_1, z_2 \in Z$. Let $\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\lambda}}(y), \mu_{\tilde{q}}(t), \mu_{\tilde{\sigma}}(u)$ denote the membership function respectively of arrival rate, service rate, breakdown rate and variance.

Then we have

$$\tilde{\lambda} = \left\{\left(x, \mu_{\tilde{\lambda}}(x)\right)/x \in X\right\}$$

$$\tilde{\mu} = \left\{\left(y, \mu_{\tilde{\mu}}(y)\right)/y \in Y\right\}$$

$$\tilde{q} = \left\{\left(t, \mu_{\tilde{q}}(t)\right)/t \in T\right\}$$

$$\tilde{\sigma} = \left\{\left(u, \mu_{\tilde{\sigma}}(u)\right)/u \in U\right\}$$

where X, Y, T, U are crisp universal set of arrival rate, service rate, breakdown rate and variance respectively. Let P(x, y, t, u) denote the system performance measures of interest.

Clearly when $\tilde{\lambda}, \tilde{\mu}, \tilde{q}$ and $\tilde{\sigma}$ are fuzzy numbers $\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})$ will be fuzzy as well. On the basis of Zadeh's extension principle the membership function of performance measure $\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})$ is defined as

$$\mu_{\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})}(Z)$$

$$= \sup_{\substack{x \in X \\ t \in T}} \min\left\{\tilde{\lambda} = \left\{\left(\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\mu}}(y), \mu_{\tilde{q}}(t), \mu_{\tilde{\sigma}}(u)\right)/\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{t}, \tilde{u})\right\}\right\}$$

without loss of generality let us assume the performance measure of interest is the expected number of customers in queue $E(N_S)$. The membership function for the performance measures is

$$\sup_{\substack{x \in X \\ t \in T}} \min\left\{\tilde{\lambda} = \left\{\left(\mu_{\tilde{\lambda}}(x), \mu_{\tilde{\mu}}(y), \mu_{\tilde{q}}(t), \mu_{\tilde{\sigma}}(u)\right)/\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{t}, \tilde{u})\right\}\right\}$$

where $\rho = xy_2$; $\mu_R = y_1$, $\mu_S = y_2$, $\sigma_R = y_1$, $\sigma_S = u_2$.

Although the membership functions are theoretically correct they are not in the usual forms. A pair of MINLP is developed to find the α cut of $\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})$ based on Zadeh's Principle. The solution procedure is given in the next section.

III. SOLUTION PROCEDURE

One approach is to construct the membership $\mu_{\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})}(Z)$ on the basis of deriving the α cuts. Denote the α cuts as follows.

$$\lambda_\alpha = \left[X_\alpha^L, X_\alpha^U\right] = \left[\min_{x \in X}\left\{x/\mu_{\tilde{\lambda}}(x) \geq \alpha\right\}, \max_{x \in X}\left\{x/\mu_{\tilde{\lambda}}(x) \geq \alpha\right\}\right]$$

$$\mu_\alpha = \left[Y_\alpha^L, Y_\alpha^U\right] = \left[\min_{y \in Y}\left\{y/\mu_{\tilde{\mu}}(y) \geq \alpha\right\}, \max_{y \in Y}\left\{y/\mu_{\tilde{\mu}}(y) \geq \alpha\right\}\right]$$

$$q_\alpha = \left[T_\alpha^L, T_\alpha^U\right] = \left[\min_{t \in T}\left\{t/\mu_{\tilde{q}}(t) \geq \alpha\right\}, \max_{t \in T}\left\{t/\mu_{\tilde{q}}(t) \geq \alpha\right\}\right]$$

$$\sigma_\alpha = \left[U_\alpha^L, U_\alpha^U\right] = \left[\min_{u \in U}\left\{u/\mu_{\tilde{\sigma}}(u) \geq \alpha\right\}, \max_{u \in U}\left\{u/\mu_{\tilde{\sigma}}(u) \geq \alpha\right\}\right]$$

These intervals indicate that the group of arrival rate, service rate, rate and variance lie at the possibility level α. Note that λ_α, μ_α, q_α and σ_α are crisp set rather than fuzzy set. By the convexity of fuzzy number the bounds of these intervals are functions, of α and can be obtained by

$$X_\alpha^L = \min \mu_{\tilde{\lambda}}^{-1}(\alpha) \qquad X_\alpha^U = \max \mu_{\tilde{\lambda}}^{-1}(\alpha)$$

$$Y_\alpha^L = \min \mu_{\tilde{\mu}}^{-1}(\alpha) \qquad Y_\alpha^U = \max \mu_{\tilde{\mu}}^{-1}(\alpha)$$

$$T_\alpha^L = \min \mu_{\tilde{q}}^{-1}(\alpha) \qquad T_\alpha^U = \max \mu_{\tilde{q}}^{-1}(\alpha)$$

$$U_\alpha^L = \min \mu_{\tilde{\sigma}}^{-1}(\alpha) \qquad U_\alpha^U = \max \mu_{\tilde{\sigma}}^{-1}(\alpha)$$

The membership function of $\tilde{P}(\tilde{\lambda}, \tilde{\mu}, \tilde{q}, \tilde{\sigma})$ is also parameterized by α. Consequently we can use its α-cut to construct the corresponding membership function. Consider the membership function of the expected number of customers in queue $\tilde{E}(N_S)$.

$\mu_{\tilde{E}(N_S)}$ is the minimum of $\mu_{\tilde{\lambda}}(x)$, $\mu_{\tilde{\mu}}(y)$, $\mu_{\tilde{q}}(t)$ and $\mu_{\tilde{\sigma}}(u)$. We need at least one of the following cases held such that Z satisfies $\mu_{\tilde{E}(N_S)}(Z) = \alpha$.

Case (i)

$$\{\mu_{\tilde{\lambda}}(x) = \alpha,\ \mu_{\tilde{\mu}}(y) \geq \alpha,\ \mu_{\tilde{q}}(t) \geq \alpha \text{ and } \mu_{\tilde{\sigma}}(u) \geq \alpha\}$$

Case (ii)

$$\{\mu_{\tilde{\lambda}}(x) \geq \alpha,\ \mu_{\tilde{\mu}}(y) = \alpha,\ \mu_{\tilde{q}}(t) \geq \alpha \text{ and } \mu_{\tilde{\sigma}}(u) \geq \alpha\}$$

Case (iii)

$$\{\mu_{\tilde{\lambda}}(x) \geq \alpha,\ \mu_{\tilde{\mu}}(y) \geq \alpha,\ \mu_{\tilde{q}}(t) = \alpha \text{ and } \mu_{\tilde{\sigma}}(u) \geq \alpha\}$$

Case (iv)

$$\{\mu_{\tilde{\lambda}}(x) \geq \alpha,\ \mu_{\tilde{\mu}}(y) \geq \alpha,\ \mu_{\tilde{q}}(t) \geq \alpha \text{ and } \mu_{\tilde{\sigma}}(u) = \alpha\}$$

To find the membership function $\mu_{\tilde{E}(N_S)}(Z)$ it suffices to find Z^L_α and Z^U_α of the α cuts of $\mu_{\tilde{E}(N_S)}(Z)$. Since the requirement of $\mu_{\tilde{\lambda}}(x) = \alpha$ can be represented by $X = X^L_\alpha$ or $X = X^U_\alpha$ this can be formulated as the constraint of $X = \gamma_1 X^L_\alpha + (1-\gamma_1)X^U_\alpha$, where $\gamma_1 = 0$ or 1. Similarly others can be formulated as follows.

$Y = \gamma_2 Y^L_\alpha + (1-\gamma_2)Y^U_\alpha$, where $\gamma_2 = 0$ or 1 for $\mu_{\tilde{\mu}}(y) = \alpha$.

$T = \gamma_3 T^L_\alpha + (1-\gamma_3)T^U_\alpha$, where $\gamma_3 = 0$ or 1 for $\mu_{\tilde{q}}(t) = \alpha$ and

$U = \gamma_4 Y^L_\alpha + (1-\gamma_4)Y^U_\alpha$, where $\gamma_4 = 0$ or 1 for $\mu_{\tilde{\sigma}}(u)$.

Moreover from the definition of $\lambda_\alpha, \mu_\alpha, q_\alpha, \sigma_\alpha$. We can now have $X \in [X^L_\alpha, X^U_\alpha]$, $Y \in [Y^L_\alpha, Y^U_\alpha]$, $T \in [T^L_\alpha, T^U_\alpha]$, $U \in [U^L_\alpha, U^U_\alpha]$.

Consequently we can find the lower bound and upper bounds of the α cuts of $\mu_{\tilde{E}(N_S)}(Z)$ as follows.

$$[E(N_S)]^{L_1}_\alpha = \min\left\{Z = \frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that

$X = \gamma_1 X^L_\alpha + (1-\gamma_1)X^U_\alpha$, $Y^L_\alpha \leq Y \leq Y^U_\alpha$; $T^L_\alpha \leq T \leq T^U_\alpha$; $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_1 = 0$ or 1.

$$[E(N_S)]^{L_2}_\alpha = \min\left\{Z = \frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y = \gamma_2 Y^L_\alpha + (1-\gamma_2)Y^U_\alpha$, $T^L_\alpha \leq T \leq T^U_\alpha$; $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_2 = 0$ or 1.

$$[E(N_S)]^{L_3}_\alpha = \min\left\{Z = \frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y^L_\alpha \leq Y \leq Y^U_\alpha$;
$T = \gamma_3 T^L_\alpha + (1-\gamma_3)T^U_\alpha$, $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_3 = 0$ or 1.

$$[E(N_S)]^{L_4}_\alpha = \min\left\{Z = \frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y^L_\alpha \leq Y \leq Y^U_\alpha$;
$T^L_\alpha \leq T \leq T^U_\alpha$; $U = \gamma_4 U^L_\alpha + (1-\gamma_4)U^U_\alpha$, $\gamma_4 = 0$ or 1.

$$[E(N_S)]^{U_1}_\alpha = \max\left\{\frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X = \gamma_1 X^L_\alpha + (1-\gamma_1)X^U_\alpha$, $Y^L_\alpha \leq Y \leq Y^U_\alpha$;
$T^L_\alpha \leq T \leq T^U_\alpha$; $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_1 = 0$ or 1.

$$[E(N_S)]^{U_2}_\alpha = \max\left\{\frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y = \gamma_2 Y^L_\alpha + (1-\gamma_2)Y^U_\alpha$,
$T^L_\alpha \leq T \leq T^U_\alpha$; $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_2 = 0$ or 1.

$$[E(N_S)]^{U_3}_\alpha = \max\left\{\frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y^L_\alpha \leq Y \leq Y^U_\alpha$; $T = \gamma_3 T^L_\alpha + (1-\gamma_3)T^U_\alpha$, $U^L_\alpha \leq U \leq U^U_\alpha$; $\gamma_3 = 0$ or 1.

$$[E(N_S)]^{U_4}_\alpha = \max\left\{\frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)]}{2(1-\rho(1+ty_1))}\right\}$$

such that $X^L_\alpha \leq X \leq X^U_\alpha$; $Y^L_\alpha \leq Y \leq Y^U_\alpha$;
$T^L_\alpha \leq T \leq T^U_\alpha$; $U = \gamma_4 U^L_\alpha + (1-\gamma_4)U^U_\alpha$, $\gamma_4 = 0$ or 1.

These models are MINLP with 0 – 1 variables. The crisp interval $[E(N_S)^L_\alpha, E(N_S)^U_\alpha]$ represents the α-cut of $\tilde{E}(N_S)$.

According to Zadeh's Principle $\tilde{E}(N_S)$ is a fuzzy number that posses convexity. Therefore for two values α_1 & α_2 such that $0 < \alpha_2 < \alpha_1 \leq 1$ we have $\tilde{E}(N_S)^L_{\alpha_1} \geq \tilde{E}(N_S)^L_{\alpha_2}$ and

$\tilde{E}\left(N_S\right)_{\alpha_1}^{U} \geq \tilde{E}\left(N_S\right)_{\alpha_2}^{U}$. In other words $E\left(N_S\right)_{\alpha}^{L}$ is non decreasing and $E\left(N_S\right)_{\alpha}^{U}$ is non increasing with respect to α which assures the convexity of $\tilde{E}\left(N_S\right)$. If both $E\left(N_S\right)_{\alpha}^{L}$ and $E\left(N_S\right)_{\alpha}^{U}$ are invertible with respect to α then a left shape function $L_S(Z) = \left[E\left(N_S\right)_{\alpha}^{L}\right]^{-1}$ and a right shape function $R_S(Z) = \left[E\left(N_S\right)_{\alpha}^{U}\right]^{-1}$ can be obtained. From this the membership for $\mu_{\tilde{E}(N_S)}$ is considered as

$$\mu_{\tilde{E}(N_S)}(Z) = \begin{cases} L_S(Z) & , E\left(N_S\right)_{\alpha=0}^{L} \leq Z \leq E\left(N_S\right)_{\alpha=1}^{L} \\ 1 & , E\left(N_S\right)_{\alpha=1}^{L} \leq Z \leq E\left(N_S\right)_{\alpha=1}^{U} \\ R_S(Z) & , E\left(N_S\right)_{\alpha=1}^{U} \leq Z \leq E\left(N_S\right)_{\alpha=0}^{U} \end{cases}$$

Although the exact function is not known the set of intervals $\left\{\left[E\left(N_S\right)_{\alpha}^{L},\ E\left(N_S\right)_{\alpha}^{U}\right] / \alpha \in [0, 1]\right\}$ reveals the shape of $\mu_{\tilde{E}(N_S)}$.

IV. NUMERICAL EXAMPLE

Consider a computer communication network where the message stream arrival rate $\tilde{\lambda} = (1, 2, 3, 4)$ the mean time of message generating (service) $\tilde{\mu}_S = (5, 6, 7, 8)$; the variance of message generating (service) $\tilde{\sigma}_S^2 = (1.5, 2.5, 3.5, 4.5)$; the breakdown rate $\tilde{q} = (0.01, 0.02, 0.03, 0.04)$; the mean time to repair $\tilde{\mu}_R = (9, 10, 11, 12)$; the variance of repair time $\tilde{\sigma}_R^2 = (5.5, 6.5, 7.5, 8.5)$ under N = 3 policy where $\tilde{\lambda}, \tilde{\mu}_R, \tilde{\sigma}_R^2, \tilde{\mu}_S, \tilde{\sigma}_R^2, \tilde{q}$ are all fuzzy numbers.

$$\tilde{E}\left(N_S\right) = \frac{N-1}{2} + \rho(1+ty_1) + \frac{x^2\left[(1+ty_1)^2(y_2^2+y_2)+ty_2(y_1^2+y_1)\right]}{2(1-\rho(1+ty_1))}$$

$$\left[\lambda^L,\ \lambda^U\right] = [1 + \alpha, 4 - \alpha] \quad \left[\mu_R^L,\ \mu_R^U\right] = [9 + \alpha, 12 - \alpha]$$

$$\left[\mu_S^L,\ \mu_S^U\right] = [5 + \alpha, 8 - \alpha] \quad \left[\tilde{\sigma}_R^{2L},\ \tilde{\sigma}_R^{2U}\right] = [5.5 + \alpha, 8.5 - \alpha]$$

$$\left[\tilde{\sigma}_R^{2L},\ \tilde{\sigma}_R^{2U}\right] = [1.5 + \alpha, 4.5 - \alpha]$$

$$\left[q^L,\ q^U\right] = [0.01+0.01\alpha, 0.04- 0.01\alpha]$$

TABLE I

α	$E\left(N_S\right)^L$	$E\left(N_S\right)^U$
0	2.42644	14.04724
0.1	2.68234	13.44830
0.2	2.94099	12.87008
0.3	2.20365	12.31184
0.4	3.47129	11.77281
0.5	3.74466	11.25229
0.6	4.02438	10.74954
0.7	4.31105	10.26388
0.8	4.60519	9.79462
0.9	4.90731	9.34117
1	5.21791	8.63668

V. CONCLUSION

This paper applies the concept of a cuts and Zadeh's Extension Principle to construct the membership functions of expected number of customers in queue using MINLP. Since the performance measure is expressed by the membership function rather than by a crisp value it maintains the fuzziness of input information and the results can be used to represent the fuzzy system more accurately.

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